Your guide to writing a Senior Capstone Major Paper
South Dakota State University, Department of Mathematics and Statistics

Chapter 1. Parts of Your Paper

1. Title Page

The **Title Page** includes the following information, centered on the page

Your Name

Title of your paper
Advised by:

Senior Seminar
Department of Mathematics and Statistics
South Dakota State University
Semester and year of submission (i.e. Spring 2006)

You may be creative with the title page if you like, with fonts or pictures, as long as all of the above information is included.

2. Abstract Page

The **Abstract** is a one to two paragraph introduction to your paper. This includes no background material- it is essentially a table of contents of your paper, written in paragraph form. Do not use "I." Begin by identifying the subject of your paper, and then summarize the paper. This will replace a "table of contents" in your paper. This page is not numbered. (LaTex users should place their abstract in the \begin{abstract} … \end{abstract} environment.)

3. Body of the Paper

This section should be ten to fifteen pages long (longer is fine, if your paper contains lots of graphics). Page one should include the title of your paper, followed by the paper itself. Each page (except page one) should be numbered in the upper right hand corner or the bottom right hand corner of the paper. Note that the first page number you will label is a "2" on the second page of the body of your paper. (To do this in Microsoft Word, you should insert a section break after your abstract page. Then begin numbering in the body of the paper, with "different first page.")

You should have one inch margins and double space your paper. Papers will be photocopied for readers, so they will receive black and white copies only. If you want to use colors in your paper, you must provide 2 extra copies of the page(s) which have color. More details on writing mathematically will be given later. Section headings within the paper should be numbered and in bold face on a separate line using the same font size as the regular text. An example is shown throughout this guide. (LaTeX users should use the \section command.)

4. Bibliography

Your Bibliography MUST satisfy these requirements:

- The bibliography is at the end of your paper.
- This page is not numbered.
- Items in the bibliography are alphabetized and numbered; NOT separated by type of source.
- Entries in the bibliography are consistently formatted
- Titles are italicized, NOT underlined.
- Items retrieved from Library databases (Jstor, MathSciNet,etc.) are referenced using the same format as those retrieved in paper format (no reference is made to database).

Our format is as follows. Please do not underline anything.

Books:

1. Enzensberger, Hans Magnus. *The Number Devil: A Mathematical Adventure*. New York: Henry Holt and Company, LLC, 1997.

Journal Article:

2. Flint, Donna. "Nonlinear Ordinary Differential Equations with Discontinuities." *Libertas Mathematica* 22 (2002) 153-166.

Interview:

3. Kemp, Daniel. Interview by author. September 23, 2002, Brookings, South Dakota.

Internet site:

4. O'Connor, J.J. and E F Robertson. "A History of Pi." St. Andrews, UK: 2001. http://www-gap.dcs.st-and.ac.uk/~history/HistTopics/Pi_through_ the_ages.html (accessed January 5, 2003).

Source: http://www.press.uchicago.edu/books/turabian/turabian_citationguide.html

Citing sources in the body of your paper: When referring to bibliography items in the paper itself, reference with the number of the citation in square brackets [ ] . For example:

Most people agree that the first theoretical calculation of pi was carried out by Archimedes of Syracuse (287-212 BC) [4].

The square root of 16 is equal to 4 [1, p 80]. If multiple pages are cited use, for example [1, pp 80-83].

(Note: you probably would not need to cite well-known information such as square root of 16 is equal to 4, but if you did, this is how you would do it!)

5. Short Biographical Statement

Write a short (2-3 paragraph) biographical statement about yourself- in third person. This will be on the last page of your paper- either at the bottom of your bibliography or on a separate page. If a separate page, this page is not numbered.

Ex: Jane Smith is from Minnesota and is currently an undergraduate at South Dakota State University. She wrote this paper as part of the Senior Capstone requirements. She will complete her B.S. in Mathematics in May 2003 and attend graduate school to pursue a Ph.D. in Mathematics. Jane hopes to become a College Professor.

Jane has wanted to be a mathematician since third grade when she first learned about fractions. While at SDSU, Jane served as president of the Math club and treasurer of the Chess club. In addition, she was first runner up in the 2003 Miss SDSU contest as well as the high point scorer in the 2004 Putnam Exam.

Jane would like to live in a big city, teaching at a large University. She hopes to someday be on ESPN in the World Championship of Chess Tour. As a College Professor, she believes she will have oodles of time to refine her chess game, while her graduate students do her teaching, grading and research for her.

Chapter 2. Writing Mathematics in the Body of Your Paper

Several suggestions for writing mathematics are included below. However, you will find that your best ideas come from journal articles you have read during your research. Use these articles not only for the information you can glean, but also as models for your own writing.

Starting to write is often difficult. Essentially, you should just dig in and start writing. It is not necessary to start with your first paragraph. Some authors start with their major proof; the paper then grows around this proof. Write your initial draft with the clear understanding that ***everything*** in that paper will be rewritten at least once. Clarity is achieved only after many re-writes.

1. Identify your audience

In this case, your audience is other mathematics majors with similar math backgrounds. You should be familiar with their course background so that you can decide where you should give detailed information, and where you can skip the basic details. (For example- you can assume everyone knows how to find a derivative, but you can't assume everyone knows the definition of a group.) Though you are graded by your professors, your audience should be those in your class.

2. Write for your audience

When explaining your topic, use proper grammar, spelling, and punctuation as you would in any writing. However, simple declarative sentences are usually best in mathematical writing. Remember that the topic itself may be difficult to understand- using complex sentences will only add to the difficulty.

Write your paper so that it is self-contained. Your reader should be able to fully understand your topic without referring to other sources. Your paper should include three major parts: An introduction, explanation of your topic, and a summary of the paper.

3. Include Some Mathematics!

Your paper should include some upper level mathematics. This should be mathematics ***beyond*** any course you have taken. If your paper does not include some advanced mathematics, you will be required to rewrite it. Your advisor is the best guide in determining whether you have enough advanced math to make your paper qualify as a senior paper.

One way to quantify this is to say that your paper must include a proof.

4. Use symbols well and consistently

If standard symbols and notation are already available for your topic, use those, explaining any with which your audience might not be familiar. If an idea can be expressed easily with a verbal description- do so- only use symbols if it will SIMPLIFY your explanation. If you must make up your own notation, use notation which is reasonable and be sure to explain what is represented. Use subscripts when needed, but not excessively. Choose symbols which are consistent with mathematical convention:

a,b,c	real-valued constants
f,g,h	functions
p,q	polynomial functions
x,y,z,w	real-valued variables
k,n,m,p,q	integer-valued variables
$\boldsymbol{u,v}$	vectors

Note about typesetting: Letters which represent mathematical objects (variables, sets, etc.) are always italicized- whether in an equation or in the text, while numbers are never italicized. One way to help you check this is to notice that the format of a variable must match, no matter where it is in your paper. Equation editor uses italics for variables, thus you must use them when you use that variable without equation editor also. You should also use italics when referring to the x-axis and y-coordinate.

5. Cite your Sources

You must cite your source whenever you make a statement which is not your own work: state a theorem, prove a theorem, or state a fact. This means to write the number of the Bibliographic item from which you obtained that information. You may write the number only or the number and page; either is appropriate. Notice that the citation is given at the end of the idea being cited when it is in a paragraph, or right before the statement of a theorem.

Examples:
Most people agree that the first theoretical calculation was carried out by Archimedes of Syracuse (287-212 BC) [4].
Theorem [3, p 27] There does not exist a rational number, r, such that $r^2 = 2$.

Citing a source does not allow you to use this idea without understanding and rewriting in your own words. Remember what you learned in your English classes about what constitutes plagiarism.

Very quick summary of plagiarism

OK: You put a statement or cleverly worded phrase in quotes and cite it. Quotes should be restricted to a few words, or possibly a few sentences. If you quote entire mathematical ideas or proofs, you are not satisfying the requirements of this paper- that you read, understand, and interpret a mathematical idea.

OK: You use an idea from your source, but have come to your own understanding of the idea and totally rewritten it in your own words. You still need to cite the source.

NOT OK: You copy an entire idea and key phrases from your source, yet change a few words. This is sometimes called paraphrasing, but if key phrases, wording and/or sentence structure remains intact from your source, it is really plagiarism. EVEN IF YOU CITE THE SOURCE!

Here are some websites with some examples to help you decide if you are plagiarizing:
https://my.hamilton.edu/academics/resource/wc/Using_Sources.PDF (Hamilton College)
http://oregonstate.edu/admin/stucon/plag.htm (Oregon State U.)
http://www.ehow.com/how_9265_avoid-plagiarism-research.html

However…If you have devised a theorem, derived a formula, or written a proof without using a source (perhaps with guidance from your advisor), make that clear in your paper. That is, don't make the reader wonder if you just forgot to cite your source. Examples:

> **Theorem (Flint)**: *The revised Runge-Kutta Method provides a convergent method to approximate the solution of the differential inclusion*.

OR

> Combining ideas covered in this paper, it is now possible to prove the following new theorem.

6. With a clear voice

You should write with short uncomplicated sentences. Remember that your material is already difficult for your reader; don't complicate it by using fancy sentence structure.

Mathematics is a very precise language. Words which may seem interchangeable in everyday English are not interchangeable in mathematics. For example, "find the function *f*…" or "find a function *f*…" Avoid using pronouns, particularly the word "it" and never use the same word to mean two different things- add some kind of clarifier to distinguish between them.

Remember that this is an expository paper (different from homework problems). Every word, symbol and equation is part of a sentence and every sentence is part of a paragraph. Sentences begin with a word (not a symbol, number or logical symbol).

When possible, the word "I" should be avoided. Generally, your reader is not interested in your opinion- they want to understand the topic. Often, however, authors use the royal "we" when explaining a method.

Phrases to avoid:
I think that…
It was hard for me to find…
I am sure that Newton thought…
I believe…
I thought that I would…
I think most people would…
There are at least a million other reasons…
It is clear that…

Good phrases or words:
Therefore/ hence/ accordingly/ thus, it follows that….
Assuming that…
where M stands for…
Let M represent…
Given…
Combining equation (1) and (2) above…
Notice that/ note that/ recall …
Because, since…
We will proceed with a proof by contradiction.
Recall we are assuming that….
Equation (15) tells us that…
Later, we will need….
Start by observing that…
Suppose that…

7. Graphics and Tables

Include graphics and tables where they will enhance the paper. Be sure that your graphics and tables are clear, well labeled and necessary. (Remember that a picture is not a proof, it only illustrates your proof.) Be sure that graphics are not too cluttered. You may need to include several graphics to demonstrate progressive steps of your proof. Use color where appropriate, but be sure to include 2 extra copies of any pages that include color when submitting your paper. You should include each graphic or table on the page which discusses it. Avoid wrapping your text around a graphic or table, unless you must do this to ensure it is on the same page as the discussion. Your graphics and tables should be electronically produced using WinPlot, Maple, Excel, or some other mathematical software and should be included as part of the paper. They also may be downloaded from an internet source. Both graphics and tables should be centered, labeled, numbered, and cited below the graphic.

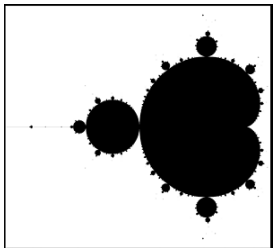

Figure 1. The Mandelbrot Set
Source: www.fractalus.com

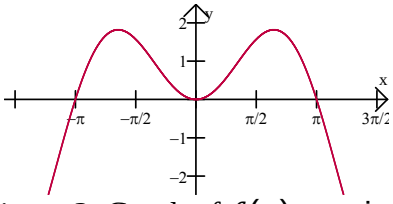

Figure 2. Graph of $f(x) = x\sin x$
Created using WinPlot

8. Words often misused:

Solve vs. Evaluate You can solve a problem or solve an equation. You can solve for a variable. You can also find a solution. However, you cannot solve a function, a variable, a point, a derivative, or an integral. You evaluate a function, a derivative at a point, or an integral.

Derive You derive a formula when you show how the formula was developed. For example, the distance formula can be derived using the Pythagorean theorem. Generally, if you are taking the derivative, it is better to say that you are going to differentiate the function.

Point vs. **Coordinate** This is a **point** : (3,2). The *x*- **coordinate** of this point is 3 and the *y*- **coordinate** is 2.

Line vs. **equation of a line** This is a **line**: ↔. This is the **equation of a line**: $y = 3x - 2$.

9. Theorems and Proofs

Theorems and their proofs should be separated from the rest of the paper with a blank line before and after the theorem. Number and/or name the theorems, using bold face type. The statement of the Theorem should be italicized. The proof should start with the word Proof in bold face type. The end of a proof should be indicated with a symbol, such as □or ◊, located on the right end of the last line of the proof, or on the blank line right after the proof is completed. An example of this format is below.
(LaTeX users should use the \begin{theorem} … \end{theorem} and \begin{proof} … \end{proof} environments with the amsthm package (see sample.tex for an example).)

Theorem (Fermat): *The Diophantine equation* $x^n + y^n = z^n$ *has no solution if* $n \geq 3$.

Proof: The proof is left for the reader as an exercise. □

The proofs should represent your understanding of a claim. Do not copy a proof from a source (even if you cite that source). Proofs in this paper should be your own work and your own explanation, based on your sources.

When writing proofs, begin by explaining what you intend to prove and what strategy you will use to prove your claim. Also, be sure to explain any notation you will be using. If your proof is long, break the proof up into several paragraphs, each one explaining a step of the proof. It may be helpful to periodically summarize what you have done and where you are going next during an especially long proof.

If your proof contains several equations, number those equations to which you will refer later, so you can refer to them easily. Be sure that your numbers are sequential throughout your entire paper. Do not number equations unless you plan to use them later in the paper. Note that an equation is part of a sentence (requiring a period). If you have an extended equation, the period goes at the end. Extended equations or statement of theorems should not be divided by a page change. A sequence of calculations should be aligned at the equal sign (in Word, this can be done in equation editor under "format"). For example:

It follows that

$$\begin{aligned} 4x^2 + 7x^2 - 2x^2 - 4 &= 9x^2 - 4 \\ &= (3x)^2 - 2^2 \\ &= (3x+2)(3x-2). \end{aligned}$$

Every statement in your proof must be mathematically correct. If you are not convinced of the validity of your statement, first convince yourself, then include in your proof those details you needed in order to be convinced.

If you make a claim that you intend to prove later, indicate that. Your reader should not spend time trying to understand why your claim is true, if you are going to justify it later.

10. Examples

Bad: So f' is at *c*.
Good: Therefore the function *f* is differentiable at *c*. *(make precise statements)*

Bad: Suppose that *x* is < 1.
Good: Suppose that *x* is less than 1. (*don't mix together words and symbols*)

Bad: It follows that *x* plus 2 is greater than 5.
Good: It follows that $x + 2 > 5$. *(sometimes symbols are better than words)*

Bad: I (or you) have now completed the proof.
Bad: In this way, one completes the proof.
Good: We have now completed the proof.
Good: This completes the proof. *(avoid I, you, and one)*

Bad: Call the smallest positive integer *z*. The smallest positive integer is 1.
(*Why not just call it 1 and be done with it?*)

Bad: Pick the smallest element of a set of *S* natural numbers. If you pick a different number from the set we are talking about, then that number will be larger than the first number that you picked.
Good: Let *s* be the least element of a set *S* of natural numbers. If *t* is an element of *S* different from *s*, then *t* is greater than *s*.
(in some cases, assigning variable names to items makes it easier to talk about them)

Bad: f is continuous.
Good: The function f is continuous on $[a, b]$. *(don't start a sentence with a symbol)*

Bad: Consider the functions $f_i, i = 1, 2, \ldots, n.$
Good: Consider the functions f_i where $i = 1, 2, \ldots, n.$
(Separate symbols with words where necessary)

Bad: *(passive)* It follows that the set X will have no element of the set Y lying in it.
Good: *(active)* Therefore no element of Y lies in X.
Good: Therefore the sets X and Y are disjoint.

Bad: We will solve the integral $\int_{-3}^{4}(3x-2)dx$ to find the area under the curve between -3 and 4.
Good: We will evaluate the integral

$$\int_{-3}^{4}(3x-2)dx$$

to find the area under the curve between -3 and 4.
(*use "solve" and "evaluate" correctly- place equations or formulas on a separate line and centered if including it in-line will make the spacing of the paper look sloppy*)

11. Other Sources you might consult
(available in the Library, or in faculty offices)

The New St. Martin's Handbook by Andrea Lunsford and Robert Connors
A Primer of Mathematical Writing by Steven G. Krantz
A Manual for Writers of Term Papers, Theses, and Dissertations (6th ed) by Kate L. Turabian
Handbook of Writing for the Mathematical Sciences by Nicholas J. Higham
Mathematical Writing by Donald E. Knuth, Tracy Larrabee, and Paul M. Roberts

Writing Math and writing in general:
http://www.fandm.edu/Departments/Mathematics/writing_in_math/guide.html
http://www.math.niu.edu/~behr/Teaching/writing.html
http://www.mth.pdx.edu/~erdman/swm/swm.pdf
http://www.swarthmore.edu/NatSci/smaurer1/WriteGuide/
http://wsuonline.weber.edu/wrh/words.htm

12. When you THINK you are done

Read your paper and ask yourself the following questions:

Did I clearly state my problem or topic?

Did I solve the problem or explain the topic I started with?

Do I fully understand all the material in this paper?

Did I clearly summarize my paper?

Did I check spelling, grammar and punctuation?

Are my symbols consistent throughout the paper?

Is the spacing consistent; symbols and equations lined up correctly?

Are my numbering systems consecutive and consistent?

Are all diagrams, tables and graphs clearly labeled?

Are all page changes smooth transitions?

Have I given credit to all my sources?

If I read this paper in 15 years, would I understand the material in the paper?

Could my friend, the math major, read and understand this paper?

Have I had sufficient feedback from my advisor?